for the computation of turning moments. Written in dimensionless form, Eq. (1) becomes

$$C_M = 3.87\,R^{-1/2} \tag{5}$$

which, for the sake of completeness, is also shown in Fig. 1 for $R \leq 3.1 \times 10^5$.

References

[1]Gregory, N., Stuart, J. T., and Walker, W. S., "On the Stability of Three-Dimensional Boundary Layers with Application to the Flow Due to a Rotating Disk," *Royal Society of London, Philosophical Transactions*, Vol. A 248, July 1955, pp. 155-199.

[2]Schlichting, H., *Boundary-Layer Theory*, McGraw-Hill, New York, 1968.

[3]Theodorsen, T. and Regier, A., "Experiments on Drag of Revolving Disks, Cylinders, and Streamline Rods at High Speeds," Rept. 793, 1944, NACA.

Velocity Distribution Equation for Laminar Unidirectional Flow in an Equilateral Triangular Conduit

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Introduction

A SIMPLE closed form solution to the Navier-Stokes equation for the laminar flow in an equilateral triangular conduit of side a is presented by Landau and Lifshitz.[1] This solution is of the form

$$u(x,y) = -\frac{2}{3^{1/2}au}\left(\frac{dp}{dz}\right) h_1\, h_2\, h_3 \tag{1}$$

where h_1, h_2, h_3 are the lengths of the perpendiculars from a given point in the triangle to its three sides, dp/dz is the constant pressure gradient, and a the length of each side of the triangle (Fig. 1). This Note shows that Eq. (1) results as a first approximation to more accurate and general variational solution of the Navier-Stokes equation. Variational solutions to the Navier-Stokes equation for other triangular geometries and rectangular geometries can also be obtained.

Variational Solution

For the coordinate system shown in Fig. 1, the Navier-Stokes equation can be written as

$$\left(\frac{\partial^2 u}{\partial x^2}\right) + \left(\frac{\partial^2 u}{\partial y^2}\right) = \frac{1}{\mu}\,\frac{dp}{dz} \tag{2}$$

The functional for the Navier-Stokes Equation (2) and for the geometry shown in Fig. 1 can be written as

$$I(u) = \int_{x=0}^{x=3^{1/2}a/2} \int_{y=-x/3^{1/2}}^{y=+x/3^{1/2}} \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + 2u\left(\frac{1}{\mu}\,\frac{dp}{dz}\right)\right] dx\,dy \tag{3}$$

$$u(x,y) = 0 \qquad \text{on the boundary} \tag{4}$$

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If the function $u(x,y)$ gives a minimum for the integral Eq. (3) it must satisfy Eq. (2).

Following the Kontorovich[2] variational method, we choose a sequence of coordinate system of functions $f_1(x,y)$, $f_2(x,y)$, ..., $f_n(x,y)$ and seek the solution of the variational problem in the form of a sum of the functions

$$u_n = \sum_{k=1}^{n} v_k(x)\, f_k(x,y) \tag{5}$$

where the coefficients $v_k(x)$ are not constants but are unknown functions of one of the independent variables that we define so that the functional $I(u)$ is extremized.

The first approximation to Eq. (5) is given by

$$u_1 = [y^2 - (x/3^{1/2})^2]\, v_1(x) \tag{6}$$

where

$$[y^2 - (x/3^{1/2})^2] = f_1(x,y)$$

The boundary condition Eq. (4), on the straight lines (Fig. 1) $y = \pm x/3^{1/2}$ are satisfied by u_1 for such a choice of $f_1(x,y)$. With this choice of u_1 Eq. (3) becomes after integration

$$I(u_1) = \frac{8\times 3^{1/2}}{405}\int_{x=0}^{x=3^{1/2}a/2} [2x^5(v_1')^2 + 10x^4 v_1 v_1' + 30x^3 v_1^2 - 15\left(\frac{1}{\mu}\,\frac{dp}{dz}\right)x^3 v_1]\,dx = \int_{x_0}^{x_1} F[x,\, v_1(x),\, v_1'(x)]\,dx \tag{7}$$

The function $v_1(x)$ is chosen so that the functional $I(u_1)$ is extremized. Hence $v_1(x)$ must satisfy Euler's equation

$$F_{v_1} - \frac{d}{dx}\left(F_{v_1'}\right) = 0 \tag{8}$$

i.e.,

$$x^2 v_1'' + 5x\, v_1' - 5v_1 = -\left(\frac{15}{4}\right)\left(\frac{1}{\mu}\right)\left(\frac{dp}{dz}\right) \tag{9}$$

The solution of Eq. (9) is

$$v_1(x) = Ax + Bx^{-5} + \frac{3}{4}\left(\frac{1}{\mu}\,\frac{dp}{dz}\right) \tag{10}$$

At $x=0$, $v_1(x)$ must be finite

$$\text{At } x = \frac{3^{1/2}}{2}a,\; v_1(x) = 0 \tag{11}$$

Therefore,

$$v_1(x) = \frac{3^{1/2}}{2a}\left(\frac{1}{\mu}\,\frac{dp}{dz}\right)\left(\frac{3^{1/2}a}{2} - x\right) \tag{12}$$

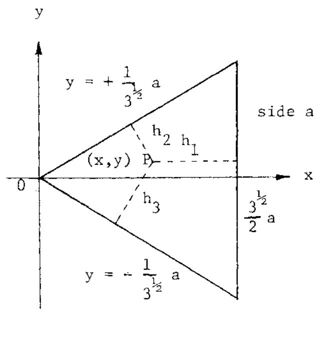

Fig. 1 Coordinates for an equilateral triangular cross section.

Eq. (6) with Eq. (12) becomes

$$u_1 = -\frac{2}{3^{1/2} a\mu}\left(\frac{dp}{dz}\right)\left[\frac{3^{1/2} a}{2} - x\right]$$

$$\left[\frac{1}{2}x - \frac{3^{1/2}}{2}y\right]\left[\frac{1}{2}x + \frac{3^{1/2}}{2}y\right]$$

$$= -\frac{2}{3^{1/2} a\mu}\left(\frac{dp}{dz}\right) h_1 h_2 h_3 \qquad (13)$$

which is same as Eq. (1) where

$$h_1 = \left(\frac{3^{1/2}}{2} a - x\right)$$

$$h_2 = \left(\frac{1}{2}x - \frac{3^{1/2}}{2}y\right)$$

$$h_3 = \left(\frac{1}{2}x + \frac{3^{1/2}}{2}y\right)$$

If a more exact answer is required, the solution may be sought in the form

$$u_2 = \left[y^2 - \left(\frac{x}{3^{1/2}}\right)^2\right] v_1(x) +$$

$$\left[y_2 - \left(\frac{x}{3^{1/2}}\right)^2\right]^2 v_2(x) \qquad (14)$$

For this choice of u_2, Eq. (7) will take the form

$$I(u_2) = \int_{x_0}^{x_l} F[x, v_l(x), v_2(x), v_1^1(x), v_2^1(x)]\, dx \qquad (15)$$

where $v_1(x)$ and $v_2(x)$ must satisfy the system of Euler's equations

$$F_{\nu_1} - (d/dx)(F_{\nu_1}^1) = 0 \qquad F_{\nu_2} - (d/dx)(F_{\nu_2}^1) = 0 \qquad (16)$$

References

[1] Landau, L. D. and Lifshitz, E. M. *Fluid Mechanics,* Addison-Wesley, Reading, Mass., 1959, p. 58.

[2] Kantorovich, L. V. and Krylov, V. I. *Approximate Methods of Higher Analysis,* Interscience, New York, 1958.

Errata

Coupled Pitch and Heave Porpoising Instability in Hydrodynamic Planing

Peter R. Payne*
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[J. Hydronautics **8,** 58-71 (1974)]

A SHORT portion of the paper was garbled when being paged from galley proofs. It appears on the right-hand side of page 8, the first complete paragraph down, and should read:

"The Day and Haag tests involved, essentially, varying the c.g. position at a given towing speed until the model porpoised with a +2° amplitude. Their raw data is plotted in Fig. 19.

"If one were unaware of the previous literature, one would be tempted to conclude from Fig. 19 that the model is unstable when the c.g. is aft of 4.0 in., and that an increased buoyancy contribution (greater weight at a given speed, or less speed) increases the stable range somewhat. One would then present the data as in Fig. 20, suspecting that the trends were all quite regular, and that the two highest speed points for Δ = 1.023 lb were somewhat in error."

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Index categories: Marine Hydrodynamics, Vessel and Control Surface; Marine Vessel Trajectories, Stability, and Control.

Announcement: 1974 Author and Subject Indexes

The indexes of the four AIAA archive journals (*AIAA Journal, Journal of Spacecraft and Rockets, Journal of Aircraft,* and *Journal of Hydronautics*) will be combined and mailed separately early in 1975. In addition, papers appearing in volumes of the *Progress in Astronautics and Aeronautics* book series published in 1974, as well as technical papers published in the 1974 issues of *Astronautics & Aeronautics,* also will be included. All subscribers to the four *Journals* are entitled to one copy of the index for each subscription which they had in 1974. All others may obtain it for $10 per copy from the Circulation Department, AIAA, Room 730, 1290 Avenues of the Americas, New York, New York 10019. **Remittance must accompany the order.**

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